Functions and the Road Coloring Problem

Estimated Time: 90 minutes

SUMMARY

This activity is adapted from the Algebra Project's Road Coloring Curriculum that introduces middle and high school kids to the concept of "function." This research-based experiential math curriculum, developed by SIUE's Dr. Greg Budzban, leads students through a long researched problem in mathematics to develop their understanding of what functions are and how they work.

WHAT YOU'LL LEARN
- To view a function as an action, then a process
- About the "Road Coloring" problem
- How to relate an experience to a mathematical concept

Materials Used	Resources Used
• 4 pieces of paper (numbered 1-4 as big as can be written on them) • 4 pieces of red ribbon, yarn, tape, or string at least 18 inches long with a "start" side and an "end" side (it needs to be an arrow) • 4 pieces of blue ribbon, yarn, tape, or string at least 18 inches long with a "start" side and an "end" side (it needs to be an arrow) • Red and Blue pens or markers	• The Algebra Project: https://algebra.org/

WHAT TO DO
1. This activity explores making a city with buildings (numbered pieces of paper) and roads (ribbons). Cities will be a collection of buildings with one-way roads connecting the buildings together. The cities must have special features, so that you can try to solve the puzzle:
    a. **Feature A** - Each building must have exactly two roads leading away from it.
    b. **Feature B** - If you start in any building, there has to be a sequence of roads that can get you to every building in your city, including back to where you started.
2. We'll refer to these specific rules by name, but here are some additional important rules that help keep you from going down the wrong path:

a. Notice that the **Feature A** only talks about how many roads leave from a building. It doesn't say anything about how many roads go to a building.
b. There can be one or more roads going to a building.
c. A road can leave from a building and come right back to the same building. This kind of road is called a "loop."
d. Both roads from a building can go to the same place.
e. Roads cannot connect to other roads.

3. Construct a city with 3 buildings using your materials in a space big enough to spread out the city and walk on. Remember to make sure that the cities satisfy Feature A and B listed above. Draw the city on a separate sheet of paper with circles with numbers in them for buildings and arrows for your roads. It doesn't matter which colors the roads are right now.
   a. Does the city satisfy both features?
   b. Try building and drawing a city with 4 buildings.
   c. Try building and drawing some other 3 and 4 building cities.

4. Next, we will "color" our roads. Once we color the roads properly, we can take a tour of our city, so we call these cities "touring-cities." Try turning all of the cities you've made so far into touring-cities. Touring-cities are those that:
   a. Satisfy both **Feature A** and **Feature B** from step 1
   b. Have exactly 1 red road and 1 blue road leading away from each building

5. To learn to use the roads, start by standing on one of the buildings. When a partner calls out "red," follow the red road leading away from the building you are in to the building it leads to, and do the same for "blue."
   a. Start at each building and figure out which set of "red" and "blue" calls will take you on a tour of each building in your city. Can you get to every building from every building?
   b. Try starting in one building and have someone else start in another building. A "red" call now has both people move on the red road from their building at the same time.

6. The Puzzle: If one person starts in each building in your city, can you come up with a set of "red" and "blue" calls that gets everyone to the same building at the same time? (See example below)
   a. What is a good name for that special set of directions that gets everyone to the same building?
   b. Have you been able to find a solution for every city? Do you think it is always possible to find a solution? Why or why not?
   c. Use one of the cities for which you were able to find a solution and then switch the color of the roads leaving from Building 1. (That is, change red to blue and blue to red.) Will the same set of directions still be a solution for the new coloring? Why or why not? (You might need to actually build the city and walk it to determine the answer.)
   d. Can you design a city and color the roads correctly so that a single command gets everyone to the same building?

Here's an example of solving the puzzle.
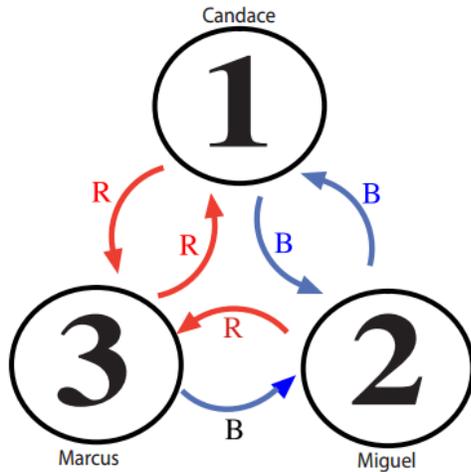
The solution to the puzzle is to call "red" and then "blue" because that leads everyone to end up at building 2 at the same time!

Name	Starting Building	After First Command: RED	After Second Command: BLUE
Candace	1	3	2
Miguel	2	3	2
Marcus	3	1	2